DSP

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For any signal, information is found in frequency component.

Frequency Component (spectrum)

- Periodic → Fourier Series
- aperiodic → general form → Fourier Transform

for discrete systems ⇒ DFT

$$X(K) = \int_{-\infty}^{\infty} x(n) e^{-jwt} dt$$

$$X(K) = \sum_{-\infty}^{\infty} x(n) e^{-jwn}$$

Periodic Sequence

$w = \frac{2\pi}{N}$ → N = No. of samples in one cycle.

$0 \longrightarrow N-1$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi Kn}{N}}$$

1st assignment

program to compute DFT

Input $x(n)$ ⎫
Output $X(K)$ ⎭ → sequence, preferred to plot it

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{n-1} x(n) \left(e^{\frac{-j2\pi}{n}} \right)^{nk}$$

$\hookrightarrow \omega_n$

$k=0 \longrightarrow X(0) = x(0)\,\omega_4^0 + x(1)\,\omega_4^0 + \cdots$

$k=1 \longrightarrow X(1) = x(0)\,\omega_4^0 + x(1)\,\omega_4^1 + x(2)\,\omega_4^2$

$$\begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(n-1) \end{bmatrix} = \begin{bmatrix} \omega_N^0 & \omega_N^0 & \cdots \\ \omega_N^0 & \omega_N^1 & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(n-1) \end{bmatrix}$$

Ex: 3-Point DFT

$x(n) = \{1, 2\}$

Soln

$$x(n) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\omega_3 = e^{-j2\pi} = \cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\omega_3^{kn} = \begin{matrix} \\ n=0 \\ 1 \\ 2 \end{matrix} \begin{matrix} k=0 \\ \end{matrix} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & \omega_3^1 & \omega_3^2 \\ 1 & \omega_3^2 & \omega_3^4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 0.5 - j\frac{\sqrt{3}}{2} \\ 0.5 + j\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$|X(K)| = \{3, 1, 1\}$$

$$\angle X(K) = \{0, -60, 60\}$$

Ex: 4 - Points DFT :-

$$x(n) = \{1, -1, 0, 1\}$$

$N = 4$

$$\times \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$W_4 = e^{-j\frac{2\pi}{4}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$W_4^{kn} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = X(K)$$

$$X(K) = \{1, 1+j2, 1, 1-2j\}$$

$$|X(k)| = \{1, \sqrt{5}, 1, \sqrt{5}\}$$

$$\angle X(K) = \{0, 63, 0, -63\}$$

3 - Points DFT

$X(n) = \{-1, 1\}$

$$X(n) = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$X(K) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\sqrt{3}/2 \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$X(K) = \left\{ 0, -\frac{3}{2} - j\frac{\sqrt{3}}{2}, -\frac{3}{2} + j\frac{\sqrt{3}}{2} \right\}$$

$$|X(K)| = \{ 0, \sqrt{3}, \sqrt{3} \}$$

$$\angle X(K) = \{ 0, 210, 150 \}$$

Continue Assignment :-

* Application depends on DSP

⇒ * General Blocks of application

⇒ * determine which blocks use DS